METHODS OF EFFICIENT MODELLING AND FORECASTING DIFFERENT SCALE ATMOSPHERIC PROCESSES

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A COMPLEX MODEL OF ATMOSPHERIC STATE

Fundamental equations of atmosphere circulation are based on the universal physics laws:

- of conservation of mass

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot V) = 0$$

- conservation of momentum
- $$\frac{DV}{Dt} + 2\Omega \times V = -\rho^{-1}\nabla p - g + \nabla \cdot (\nu \Pi)$$
- conservation of energy

$$\rho c_p \frac{DT}{Dt} - \alpha T \frac{Dp}{Dt} = \nabla\left(k\nabla T - F^{rad}\right) + Q_H$$

- conservation of scalar entities $\Re = \left(\varepsilon, k, q, q_L, q_W\right)$

$$\frac{D\Re}{Dt} = \nabla(k\nabla\Re) + Q_q$$

- and state equation $p = \rho RT$

PROBLEM DEFINITION

Therefore problem of atmosphere circulation involve systems of convection-diffusion equations as a main constituent. It is the following vector form:

$$\frac{\partial \Im}{\partial t} + v_1 \frac{\partial \Im}{\partial x_1} + v_2 \frac{\partial \Im}{\partial x_2} + v_3 \frac{\partial \Im}{\partial x_3} = F + \frac{\partial}{\partial x_1}\left(\mu_1 \frac{\partial \Im}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\mu_2 \frac{\partial \Im}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(\mu_3 \frac{\partial \Im}{\partial x_3}\right)$$

- with initial condition

$$\Im(X,0) = \eta(X) \;, \qquad 0 \le X \le L$$

- and boundary conditions

$$\Im\big|_{X=0} = \alpha(t) \;, \quad \Im\big|_{X=L} = \beta(t), \qquad t > 0$$

I. PROBLEM-SOLVING NUMERICAL PROCEDURE OF MACROSCALE FORECAST ON BASIS OF UPSTREAM FINITE-DIFFERENCE SCHEME

- Recently in weather forecast problems for numerical integration of hydro-dynamical heat/mass transmission equations more often are applied methods of a finite element and spectral methods. Yet we will consider one more finite-difference method what is explained by following reasons:
- basic concepts, underlying theory and main features for numerical applications (such as approximation, convergence and stability) are well understood and developed for finite-difference methods;
- these methods are treated universally in many applications areas;
- they allows decomposition of a complex multidimensional problem by reducing a numerical solution uniformly through spatial splitting into temporal sequence of one-dimensional problems;
- the last feature is quite appropriate for parallelizing algorithms and their efficient parallel implementation in multiprocessor computer software.

SOLVING THE THREE-DIMENSIONAL EQUATION BY OPERATOR SPLITTING

The technique is based on directional operator splitting, which results in one-dimensional advection-diffusion equations for $t \in ((k-1)\tau, k\tau]$

$$\frac{\partial \xi_k^{(1)}(t)}{\partial t} = -v_1 \frac{\partial \xi_k^{(1)}}{\partial x_1} + \frac{\partial}{\partial x_1}\left(\mu_1 \frac{\partial x_k^{(1)}}{\partial x_1}\right) + F_{x_1}, \quad \xi_k^{(1)}((k-1)\tau) = \xi_{sp}((k-1)\tau),$$

$$\frac{\partial \xi_k^{(2)}(t)}{\partial t} = -v_2 \frac{\partial \xi_k^{(2)}}{\partial x_2} + \frac{\partial}{\partial x_2}\left(\mu_2 \frac{\partial x_k^{(2)}}{\partial x_2}\right) + F_{x_2}, \quad \xi_k^{(2)}((k-1)\tau) = \xi_{sp}((k-1)\tau),$$

$$\frac{\partial \xi_k^{(3)}(t)}{\partial t} = -v_3 \frac{\partial \xi_k^{(3)}}{\partial x_3} + \frac{\partial}{\partial x_3}\left(\mu_3 \frac{\partial x_k^{(3)}}{\partial x_3}\right) + F_{x_3}, \quad \xi_k^{(3)}((k-1)\tau) = \xi_{sp}((k-1)\tau),$$

$$\xi_{sp}(k\tau) = \frac{1}{3}\sum_{i=1}^{3} \xi_k^{(i)}((k-1)\tau)$$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

- Consider the one-dimensional advection-diffusion equation

$$\frac{\partial \xi}{\partial t} + v\frac{\partial \xi}{\partial x} = \frac{\partial}{\partial x}\left(\mu \frac{\partial \xi}{\partial x}\right) + F \qquad \mu \geq 0 \qquad 0 \leq x \leq l \qquad t > 0$$

- with initial condition

$$\xi(x,0) = \eta(x) \qquad 0 \leq x \leq l$$

- and boundary conditions

$$\xi(0,t) = \alpha(t) \qquad \xi(l,t) = \beta(t) \qquad t > 0$$

- where

$$v(x,t) \quad \mu(x,t) \quad \eta(x) \quad \alpha(t) \quad \beta(t)$$

are known functions, while the function $\xi(x,t)$ is unknown.

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

- Integrating equation at x_j from t^n to t^{n+1} yields

$$\xi_j^{n+1} = \xi_j^n - \int_{t^n}^{t^{n+1}} \left[v\frac{\partial \xi}{\partial x} - \frac{\partial}{\partial x}\left(\mu \frac{\partial \xi}{\partial x}\right) - F \right]_j dt$$

- Approximating the integral on the right-hand side by the mean-value theorem, we obtain

$$\xi_j^{n+1} = \xi_j^n - \tau \left[v\frac{\partial \xi}{\partial x} - \frac{\partial}{\partial x}\left(\mu \frac{\partial \xi}{\partial x}\right) - F \right]_j^{t=\theta}$$

- where $t^n < \theta < t^{n+1}$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

- For the approximation of the derivatives $\left.(\partial\xi/\partial x)\right|_j^{t=\theta}$ and $\left.[\partial(\mu\,\partial\xi/\partial x)/\partial x]\right|_j^{t=\theta}$ we will use the following difference relations:

$$\left.\left(\frac{\partial\xi}{\partial x}\right)\right|_j^{t=\theta} = \frac{1}{h_{j-1}+h_j}\left[h_{j-1}\frac{\xi_{j+1}-\xi_j}{h_j}+h_j\frac{\xi_j-\xi_{j-1}}{h_{j-1}}\right]^{t=\theta} - \\ -\frac{h_{j-1}h_j}{6}\left(\frac{\partial^3\xi}{\partial x^3}\right)^{t=\theta}$$

$$\left.\left[\frac{\partial}{\partial x}\left(\mu\frac{\partial\xi}{\partial x}\right)\right]\right|_j^{t=\theta} = \frac{1}{h_{j-1}+h_j}\left[\left(\mu_{j+1}+\mu_j\right)\frac{\xi_{j+1}-\xi_j}{h_j} - \right. \\ \left. -\left(\mu_j+\mu_{j-1}\right)\frac{\xi_j-\xi_{j-1}}{h_{j-1}}\right]^{t=\theta} - \frac{h_j-h_{j-1}}{3}\left(\frac{\partial^3\xi}{\partial x^3}\right)^{t=\theta}$$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

The unilateral difference expressions $(\xi_{j+1} - \xi_j)/h_j$ and $(\xi_j - \xi_{j-1})/h_{j-1}$ in derivatives of order 1 and 2 will be taken at different time levels (n and $n+1$). For construction of approximations only by two points it is natural for physical reasons to have on the $(n+1)$-th layer a point x_j as central, and to select the second one from that side from where ξ is transferred by advection to the central point. In this manner we gain the following form:

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

1. for $v > 0$

$$\left(\frac{\partial \xi}{\partial x}\right)\Bigg|_j^{t=\theta} \approx \frac{1}{h_{j-1}+h_j}\left[h_{j-1}\frac{\xi_{j+1}^{n}-\xi_j^{n}}{h_j}+h_j\frac{\xi_j^{n+1}-\xi_{j-1}^{n+1}}{h_{j-1}}\right]+ \\ +\tau\frac{\partial^2\xi}{\partial t\,\partial x}\Bigg|_j^{t=\theta}$$

$$\left[\frac{\partial}{\partial x}\left(\mu\frac{\partial \xi}{\partial x}\right)\right]\Bigg|_j^{t=\theta} \approx \frac{1}{h_{j-1}+h_j}\left[\left(\mu_{j+1}+\mu_j\right)\frac{\xi_{j+1}^{n}-\xi_j^{n}}{h_j}- \right. \\ \left. -\left(\mu_j+\mu_{j-1}\right)\frac{\xi_j^{n+1}-\xi_{j-1}^{n+1}}{h_{j-1}}\right]+\tau\frac{\partial^2\xi}{\partial t\partial x}\Bigg|_j^{t=\theta}$$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

2. for $v < 0$

$$\left(\frac{\partial \xi}{\partial x}\right)\Bigg|_j^{t=\theta} \approx \frac{1}{h_{j-1}+h_j}\left[h_{j-1}\frac{\xi_{j+1}^{n+1}-\xi_j^{n+1}}{h_j}+h_j\frac{\xi_j^n-\xi_{j-1}^n}{h_{j-1}}\right]+$$

$$+\tau\frac{\partial^2\xi}{\partial t\,\partial x}\Bigg|_j^{t=\theta}$$

$$\left[\frac{\partial}{\partial x}\left(\mu\frac{\partial \xi}{\partial x}\right)\right]\Bigg|_j^{t=\theta} \approx \frac{1}{h_{j-1}+h_j}\left[\left(\mu_{j+1}+\mu_j\right)\frac{\xi_{j+1}^{n+1}-\xi_j^{n+1}}{h_j}-\right.$$

$$\left.-\left(\mu_j+\mu_{j-1}\right)\frac{\xi_j^n-\xi_{j-1}^n}{h_{j-1}}\right]+\tau\frac{\partial^2\xi}{\partial t\partial x}\Bigg|_j^{t=\theta}$$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

Difference scheme for the one-dimensional advection-diffusion problem in the following form:

- for $v > 0$

$$\frac{\xi_j^{n+1}-\xi_j^n}{\tau}+\frac{1}{h_{j-1}+h_j}\left[h_{j-1}v_j^n\frac{\xi_{j+1}^n-\xi_j^n}{h_j}+h_jv_j^{n+1}\frac{\xi_j^{n+1}-\xi_{j-1}^{n+1}}{h_{j-1}}\right]-$$

$$-\frac{1}{h_{j-1}+h_j}\left[\left(\mu_{j+1}^n+\mu_j^n\right)\frac{\xi_{j+1}^n-\xi_j^n}{h_j}-\left(\mu_j^{n+1}+\mu_{j-1}^{n+1}\right)\frac{\xi_j^{n+1}-\xi_{j-1}^{n+1}}{h_{j-1}}\right]-F_j^n=0$$

$$j=1,2,\ldots,J-1 \qquad n=0,1,\ldots N$$

$$\xi_j^0=\eta(x_j) \qquad j=0,1,\ldots,J$$

$$\xi_0^n=\alpha(t^n) \qquad \xi_J^n=\beta(t^n) \qquad n=0,1,\ldots N$$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

- for $v < 0$

$$\frac{\xi_j^{n+1}-\xi_j^n}{\tau}+\frac{1}{h_{j-1}+h_j}\left[h_{j-1}v_j^{n+1}\frac{\xi_{j+1}^{n+1}-\xi_j^{n+1}}{h_j}+h_j v_j^n\frac{\xi_j^n-\xi_{j-1}^n}{h_{j-1}}\right]-$$

$$-\frac{1}{h_{j-1}+h_j}\left[\left(\mu_{j+1}^{n+1}+\mu_j^{n+1}\right)\frac{\xi_{j+1}^{n+1}-\xi_j^{n+1}}{h_j}-\left(\mu_j^n+\mu_{j-1}^n\right)\frac{\xi_j^n-\xi_{j-1}^n}{h_{j-1}}\right]-F_j^n=0$$

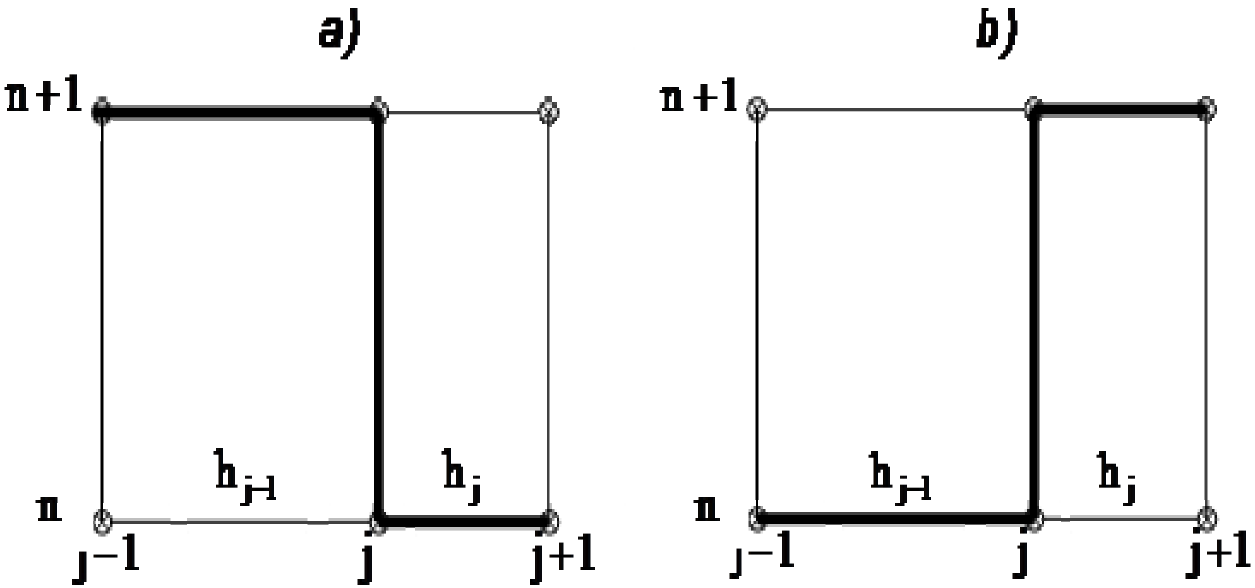
$$j = J-1, J-2, \ldots, 2, 1 \qquad n = 0, 1, \ldots N$$

$$\xi_j^0 = \eta\left(x_j\right) \qquad j = 0, 1, \ldots, J$$

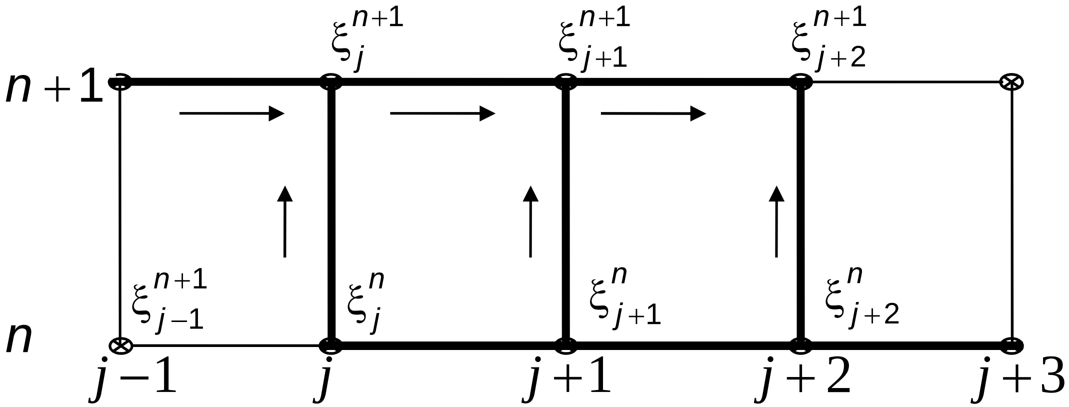
$$\xi_0^n = \alpha\left(t^n\right) \qquad \xi_J^n = \beta\left(t^n\right) \qquad n = 0, 1, \ldots N$$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

Templates of difference networks: a) of the scheme $v > 0$;
b) of the scheme $v < 0$



A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM



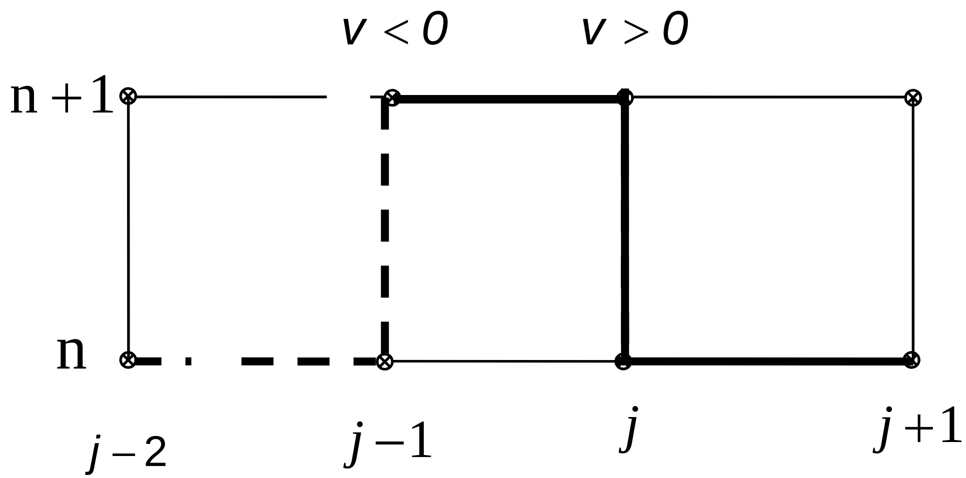
- Algorithm of solution of problem with the scheme

$$\xi_j^{n+1} = \left[p_j \xi_{j-1}^{n+1} - q_j \xi_{j+1}^{n} + (1 + q_j)\xi_j^{n} + \tau F_j^{n} \right] \Big/ (1 + p_j)$$

where

$$p_j = \frac{\tau}{h_{j-1} + h_j} \left(\frac{h_j}{h_{j-1}} v_j^{n+1} + \frac{\mu_j^{n+1} + \mu_{j-1}^{n+1}}{h_{j-1}} \right) \qquad q_j = \frac{\tau}{h_{j-1} + h_j} \left(\frac{h_{j-1}}{h_j} v_j^{n} - \frac{\mu_{j+1}^{n} + \mu_j^{n}}{h_j} \right)$$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM



Geometric illustration of the flow velocity reversal from $v < 0$ to $v > 0$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

$v > 0$

$$\xi_j^{n+1} = \left[p_j \xi_{j-1}^{n+1} - q_j \xi_{j+1}^{n} + \left(1 + q_j\right) \xi_j^{n} + \tau F_j^{n} \right] \Big/ \left(1 + p_j\right)$$

$$p_j = \frac{\tau}{h_{j-1} + h_j} \left(\frac{h_j}{h_{j-1}} v_j^{n+1} + \frac{\mu_j^{n+1} + \mu_{j-1}^{n+1}}{h_{j-1}} \right) \qquad q_j = \frac{\tau}{h_{j-1} + h_j} \left(\frac{h_{j-1}}{h_j} v_j^{n} - \frac{\mu_{j+1}^{n} + \mu_j^{n}}{h_j} \right)$$

$v < 0$

$$\xi_{j-1}^{n+1} = \left[\left(1 - s_{j-1}\right) \xi_{j-1}^{n} + s_{j-1} \xi_{j-2}^{n} - r_{j-1} \xi_j^{n+1} + f_j^{n} \right] \Big/ \left(1 - r_{j-1}\right)$$

$$r_{j-1} = \frac{\tau}{h_{j-2} + h_{j-1}} \left(\frac{h_{j-2}}{h_{j-1}} v_{j-1}^{n+1} - \frac{\mu_j^{n+1} + \mu_{j-1}^{n+1}}{h_{j-1}} \right) \qquad s_{j-1} = \frac{\tau}{h_{j-2} + h_{j-1}} \left(\frac{h_{j-1}}{h_{j-2}} v_{j-1}^{n} + \frac{\mu_{j-1}^{n} + \mu_{j-2}^{n}}{h_{j-2}} \right)$$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

$v > 0$

$$q_{j-1}^{n+1} = \frac{\left(1 + s_j\right) G_j - r_{j-1} \Im_j}{\left(1 + s_j\right)\left(1 - r_j\right) + s_j r_j}$$

$v < 0$

$$q_j^{n+1} = \frac{s_j G_j + \left(1 - r_{j-1}\right) \Im_j}{\left(1 + s_j\right)\left(1 - r_j\right) + s_j r_j}$$

where

$$G_j = \left(1 - s_{j-1}\right) q_{j-1}^n + s_{j-1} q_{j-2}^n \qquad \Im_j = \left(1 + d_j\right) q_j^n - d_j q_{j+1}^n$$

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

Numerical viscosity of difference scheme is

$$\xi = \tau v\left(\frac{v}{2} - \frac{vh_j + 2\mu}{h_{j-1} + h_j}\right)\frac{\partial^2 q}{\partial x^2} = -\mu_{eff}\,\frac{\partial^2 q}{\partial x^2}$$

If $v = 0$ or $\mu = 0$ and $h_{j-1} = h_j = const$ then $\xi = 0$

Inasmuch as

$\mu_{eff} = (vh_j + 2\mu)/(h_{j-1} + h_j) - v/2 > 0$ under $v > 0$ and $\mu > 0$

therefore $\dfrac{h_j + 2\mu/v}{h_{j-1} + h_j} \geq \dfrac{1}{2}$

scheme is stable

A FINITE-DIFFERENCE SCHEME FOR THE ADVECTION-DIFFUSION PROBLEM

The scheme stability condition is satisfied if

$$|\rho| = \frac{\left(1 + 2q_j \, sin^2 \varphi/2\right)^2 + q_j^2 \, sin^2 \varphi}{\left(1 + 2p_j \, sin^2 \varphi/2\right)^2 + p_j^2 \, sin^2 \varphi}$$

We have $q_j \leq p_j$. It means that irrespective of a ratio of grid steps h_j and τ from this expression the inequality $|\rho| \leq 1$ takes place. It follows from here the stability requirement condition of a difference grid steps h_j and τ.

A NUMERICAL EXPERIMENT

For experimental estimation of such important characteristics of difference schemes like: accuracy, stability and efficiency we consider a problem of propagation of some physical quantity $q(x_1, x_2, x_3, t)$ in viscous continuum $\mu = \{\mu_1, \mu_2, \mu_3\}$

$$\mu_1 = sin^2(x_1) \qquad \mu_2 = sin^2(x_2) \qquad \mu_3 = x_3^{2a}$$

that moves with a speed $V = \{v_1, v_2, v_3\}$

$$v_1 = \frac{1}{2} sin(2x_1) \qquad v_2 = \frac{1}{2} sin(2x_2) \qquad v_3 = a\, x_3^{2a-1}$$

A function

$$q(x_1, x_2, x_3, t) = e^{(2-b^2)t} tg\left(\frac{x_1}{2}\right) tg\left(\frac{x_2}{2}\right) sin\left[\frac{b x_3^{1-a}}{(a-1)}\right]$$

is a precise solution of the problem:

THE RESULTS OF A NUMERICAL EXPERIMENT

$$\frac{\partial q}{\partial t} + v_1 \frac{\partial q}{\partial x_1} + v_2 \frac{\partial q}{\partial x_2} + v_3 \frac{\partial q}{\partial x_3} =$$

$$= \frac{\partial}{\partial x_1}\left(\mu_1 \frac{\partial q}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\mu_2 \frac{\partial q}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(\mu_3 \frac{\partial q}{\partial x_3}\right)$$

$$t > 0 \qquad 0 \le x_1 \le \pi/2 \qquad 0 \le x_2 \le \pi/2 \qquad 0 \le x_3 \le \pi/2$$

$$q(x_1, x_2, x_3, t)\Big|_{t=0} = tg(x_1/2)\,tg(x_2/2)\,sin\left[b x_3^{1-a}(a-1)^{-1}\right]$$

$$q(x_1, x_2, x_3, t)\Big|_{x_1=0} = 0$$

$$q(x_1, x_2, x_3, t)\Big|_{x_1=\pi/2} = e^{\left(2-b^2\right)t}\, tg\left(x_2^2/2\right) sin\left[b x_3^{1-a}(a-1)^{-1}\right]$$

$$q(x_1, x_2, x_3, t)\Big|_{x_2=0} = 0$$

$$q(x_1, x_2, x_3, t)\Big|_{x_2=\pi/2} = e^{\left(2-b^2\right)t}\, tg\left(x_1^2/2\right) sin\left[b x_3^{1-a}(a-1)^{-1}\right]$$

$$q(x_1, x_2, x_3, t)\Big|_{x_3=0} = 0$$

$$q(x_1, x_2, x_3, t)\Big|_{x_3=\pi/2} = e^{\left(2-b^2\right)t}\, tg\left(x_1^2/2\right) tg\left(x_2^2/2\right) sin\left[b(\pi/4)^{1-a}(a-1)^{-1}\right]$$

THE RESULTS OF A NUMERICAL EXPERIMENT

t	Maximum fractional error of the task solution			
	$\tau = 4.7124\cdot10^{-2}$	$\tau = 2.3562\cdot10^{-2}$	$\tau = 4.7124\cdot10^{-3}$	$\tau = 4.7124\cdot10^{-4}$
0.2	0.12307	6.3636·10-2	5.2971·10-3	1.7102·10-4
0.4	0.12307	6.3636·10-2	5.2972·10-3	1.7106·10-4
0.6	0.12307	6.3636·10-2	5.2972·10-3	1.7107·10-4
0.8	0.12307	6.3636·10-2	5.2972·10-3	1.7108·10-4
1.0	0.12307	6.3636·10-2	5.2972·10-3	1.7108·10-4
1.2	0.12307	6.3636·10-2	5.2972·10-3	1.7108·10-4
1.4	0.12307	6.3636·10-2	5.2972·10-3	1.7108·10-4
1.6	0.12307	6.3636·10-2	5.2972·10-3	1.7108·10-4
1.8	0.12307	6.3636·10-2	5.2972·10-3	1.7108·10-4
2.0	0.12307	6.3636·10-2	5.2972·10-3	1.7108·10-4

II. PROBLEM-SOLVING NUMERICAL PROCEDURE OF MESOSCALE FORECAST ON BASIS OF MULTIPLE NODES INTERPOLATION TECHNIQUE

- Regional atmospheric processes are influenced by macroscale atmospheric circulation where modeling meteorological values in restricted area is considered as a part some whole with time dependent, transitional boundary conditions. To achieve demanded level of accuracy of the solutions for a model in places of heavy gradients of related functions it is often needed to have variable grid step of numerical solution for restricted terrains. However common techniques of mathematical physics often can not satisfy these requirements because of low accuracy, slow divergence and suffering from stability problems, so some dedicated numerical modeling are needed to make computational methods efficient.

INTRODUCTION

- For forecasting of values of meteorological quantities (components v_1, v_2, v_3 of velocity $\mathbf{v}$, pressure p, temperature θ, specific humidity ρ, specific liquid water content q, concentration of pollutants υ and others) of an atmosphere above the limited territory we will follow basics of the method of "unilateral influence" where results of analysis and forecast received within macroscale (hemisphere or global) model are used as boundary conditions for a regional model.

PROBLEM STATEMENT AND A METHOD OF ITS NUMERICAL SOLUTION

- Let a state of atmosphere in space $r = (\lambda, \varphi, \sigma)$ of macrospace area $G(r) \supset \overline{G}(r)$ is defined by a vector of meteorological quantities $\Re(r,t)$ of discrete values of the analysis and forecast $\Re\left(r, t^{m+1}\right) = \Re^{m+1}(r)$ received on a basis of macrospace model at the moment of time $t = t^{m+1}$ $(m = 0, 1, ..., M)$ with a step $\tau = t^{m+1} - t^{m}$.
- Then for definition of a state of atmosphere in the limited territory $\overline{G}$ at $\forall t \in \left[t^{m}, t^{m+1}\right]$ we will solve a task of following kind in vector representation:

PROBLEM STATEMENT (continuation)

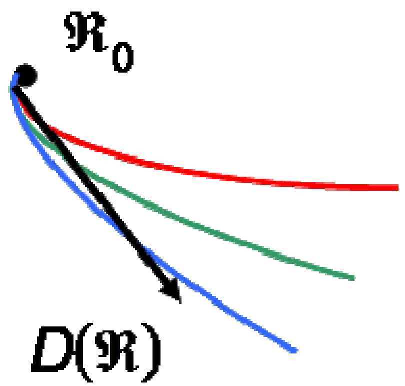
$$\frac{\partial \Re}{\partial t} = D\Re, \quad \forall t \in \left[t^{m}, t^{m+1} \right], \qquad \forall r \in \overline{G}$$

$$\Re\left(r, t^{m+1} \right) = \Re^{m+1}(r), \qquad m = 0, 1, \ldots, M$$

where

$$D\Re = \frac{1}{r\cos\varphi}\frac{\partial}{\partial\lambda}\left(\frac{\nu_1}{r\cos\varphi}\frac{\partial\Re}{\partial\lambda}\right) + \frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{\nu_2}{r}\frac{\partial\Re}{\partial\varphi}\right) + \frac{\partial}{\partial r}\left(\nu_3\frac{\partial\Re}{\partial r}\right) -$$

$$-\frac{v_1}{r\cos\varphi}\frac{\partial\Re}{\partial\lambda} - \frac{v_2}{r}\frac{\partial\Re}{\partial\varphi} - v_3\frac{\partial\Re}{\partial r} + F$$



APPROXIMATION OF DIFFERENTIAL OPERATORS BY GRID ONES

- Computation of grid values of partial derivative of the first order $\psi_i = (\partial\Re/\partial\eta)_i$ and of the second order $\zeta_i = \left(\partial^2\Re/\partial\eta^2\right)_i$ included in the differential operator D, will be performed on the basis of relations:
- first order

$$\psi_{i+1} + 2\left(1 + \frac{h_i}{h_{i-1}}\right)\psi_i + \frac{h_i}{h_{i-1}}\psi_{i-1} =$$

$$= \frac{3}{h_i}\left\{\Re_{i+1} - \left[1 - \left(\frac{h_i}{h_{i-1}}\right)^2\right]\Re_i - \left(\frac{h_i}{h_{i-1}}\right)^2 \Re_{i-1}\right\} -$$

$$- \frac{h_i h_{i-1}^2}{24}\left[1 - \left(\frac{h_i}{h_{i-1}}\right)^2\right]\frac{\partial^4\Re}{\partial\eta^4}$$

APPROXIMATION OF DIFFERENTIAL OPERATORS BY GRID ONES (continuation)

- second order

$$\frac{h_{i-1}}{h_i}\left[\frac{h_{i-1}}{h_i}\left(1-\frac{h_{i-1}}{h_i}\right)+1\right]\xi_{i+1}+$$

$$+\left(1+\frac{h_{i-1}}{h_i}\right)\left[\frac{h_{i-1}}{h_i}\left(3+\frac{h_{i-1}}{h_i}\right)+1\right]\xi_i+\left[\frac{h_{i-1}}{h_i}\left(1+\frac{h_{i-1}}{h_i}\right)-1\right]\xi_{i-1}=$$

$$=\frac{12}{h_i^2}\left[\frac{h_{i-1}}{h_i}\Re_{i+1}-\left(1+\frac{h_{i-1}}{h_i}\right)\Re_i+\Re_{i-1}\right]+$$

$$+\frac{h_i^2 h_{i-1}}{360}\left[1-\left(\frac{h_{i-1}}{h_i}\right)^2\right]\left\{5\frac{h_{i-1}}{h_i}+2\left[1-\left(\frac{h_{i-1}}{h_i}\right)^2\right]\right\}\frac{\partial^5\Re}{\partial\eta^5}$$

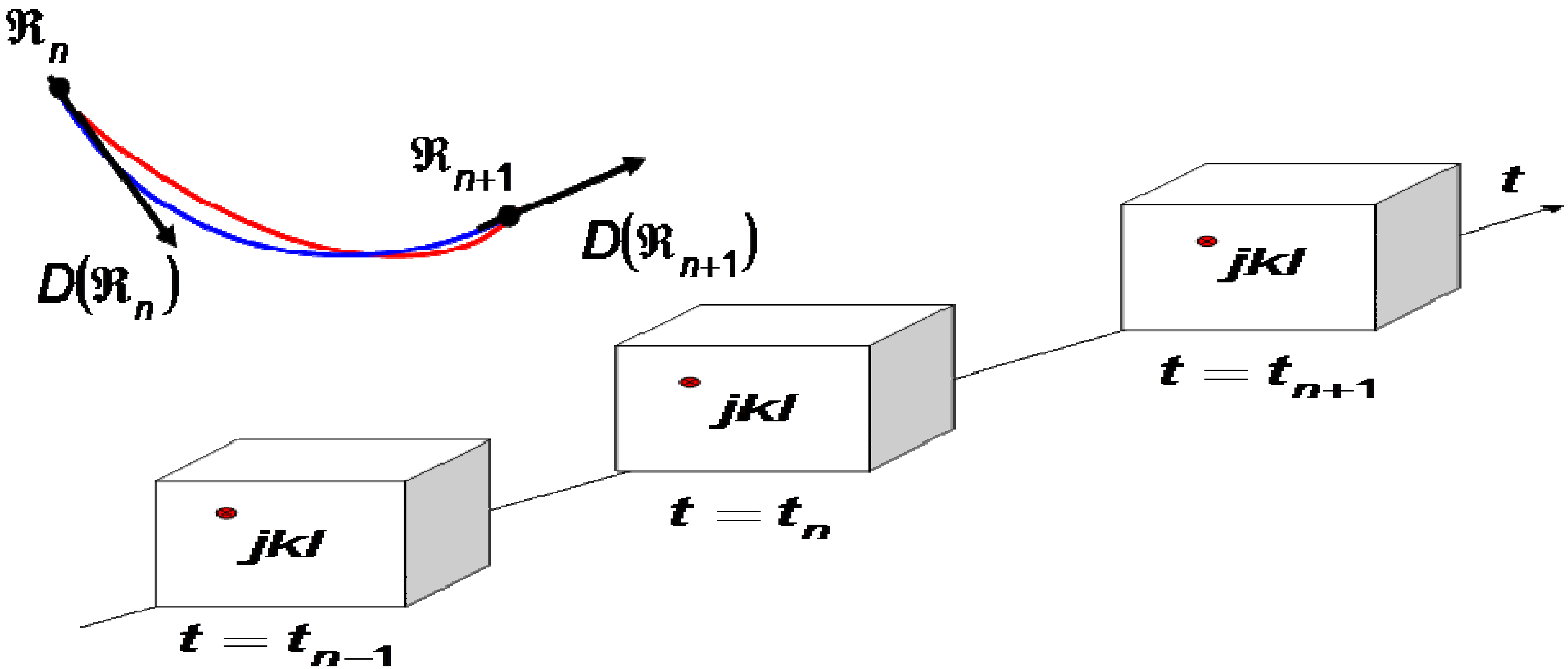
APPROXIMATION OF DIFFERENTIAL OPERATORS BY GRID ONES (continuation)

- It is obvious, that the derived relations have the third order at
- $h_i \neq h_{i-1}$ and fourth order at $h_i = h_{i-1}$. These systems are the algebraic equations with three-diagonal matrixes, so solutions can be found with boundary conditions:

- $$-\frac{h_1}{6}\left(\xi_2 - \xi_1\right) + \psi_1 + \psi_2 = 2\frac{\Re_2 - \Re_1}{h_1},$$
- $$-\frac{h_{N-1}}{6}\left(\xi_N - \xi_{N-1}\right) + \psi_{N-1} + \psi_N = 2\frac{\Re_N - \Re_{N-1}}{h_{N-1}}.$$

- The main advantage of the offered method of approximation of derivatives. As a solution of the system of algebraic equations in all points depends on values in other points, it depends on $\Re_i$ globally instead of locally that means smooth filling up and approximation of differential operators by grid ones.

PROPOSED PROBLEM-SOLVING PROCEDURE



$$\frac{\partial \Re_{jkl}}{\partial t} = \Lambda \Re^{n}_{jkl} \equiv f^{n}_{jkl} \qquad \forall t \in \left[t_n, t_{n+1} \right]$$

$$\Re_{jkl}(t_p) = \Re^{p}_{jkl} \qquad f^{p}_{jkl} = \Lambda \Re^{p}_{jkl} = \Lambda \Re_{jkl}(t_p) \qquad p = n+1,\, n\,,\, n-1$$

PROBLEM-SOLVING PROCEDURE

- After computation of values of right parts $f\left(t^{m+1}\right) = f^{m+1} = D\Re\left(t^{m+1}\right) = \Lambda\Re^{m+1}$, $m = 1,2,...,M$ in all nodes of the grid $\left(\lambda_j, \varphi_k, \sigma_l\right)$, $1 \le j \le J$, $1 \le k \le K$, $1 \le l \le L$, we will search for a solution of the problem for with the help of Hermite polynomial like above for number of points:

$$\Re(t) = \Re^m + \frac{t-t^m}{\tau}\Bigg[\ \tau f^m +$$
$$+\frac{t-t^m}{4\tau}\Bigg[\ 4\left(\Re^{m+1} - 2\Re^m + \Re^{m-1}\right) - \tau\left(f^{m+1} - f^{m-1}\right) +$$
$$+\frac{t-t^m}{4\tau}\Bigg[\ 5\left(\Re^{m+1} - \Re^{m-1}\right) - \tau\left(f^{m+1} + 8f^m + f^{m-1}\right) -$$
$$-\frac{t-t^m}{4\tau}\Bigg[\ 2\left(\Re^{m+1} - 2\Re^m + \Re^{m-1}\right) - \tau\left(f^{m+1} - f^{m-1}\right) +$$
$$+\frac{t-t^m}{4\tau}\Bigg[\ 3\left(\Re^{m+1} - \Re^{m-1}\right) - \tau\left(f^{m+1} + 4f^m + f^{m-1}\right)\Bigg]\Bigg]\Bigg]\Bigg]\Bigg]$$

PROBLEM DEFINITION

A function

$$u = \sin(\pi t + \varphi)\exp(at)$$

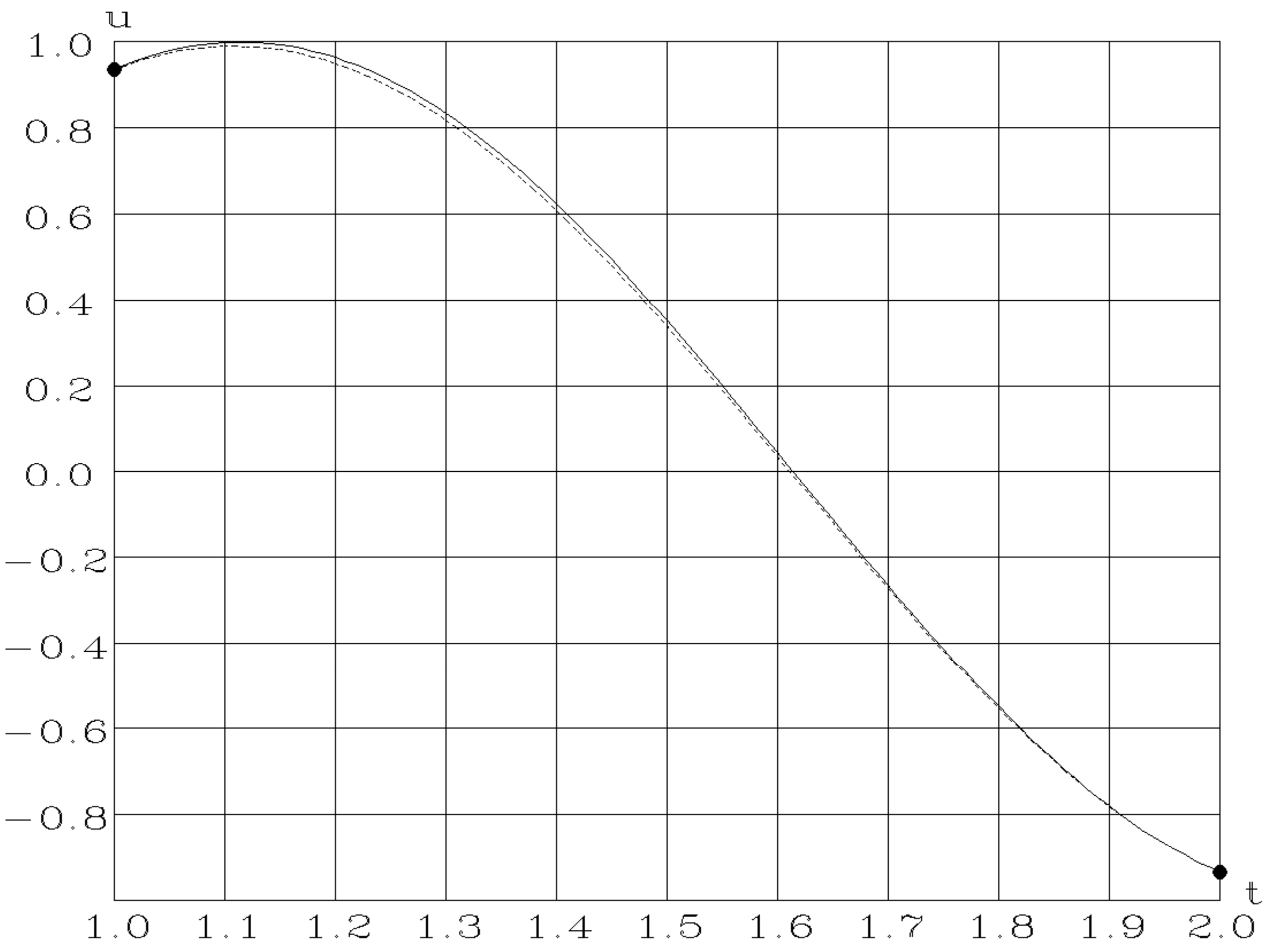
is a precise solution of the problem:

$$\frac{du}{dt} - au = \pi\exp(at)\cos(\pi t + \varphi) \qquad t \in [2, 3]$$

$$u(t_i) = u_i \ , \qquad i = 1, 2, 3 \ ,$$

where $a = 0.2, \ \varphi = 1.4$ are known constants

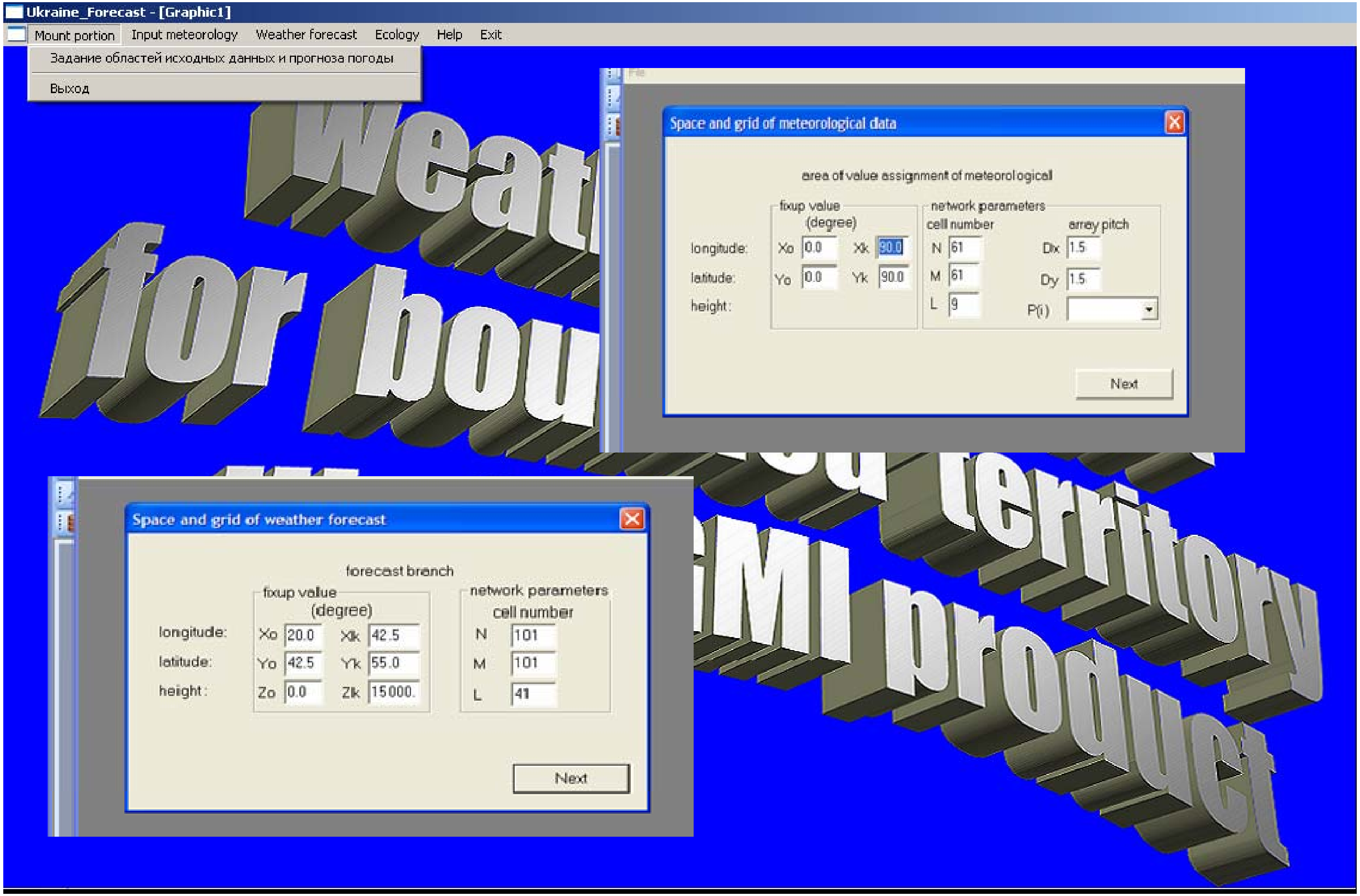
THE RESULTS OF A NUMERICAL EXPERIMENT



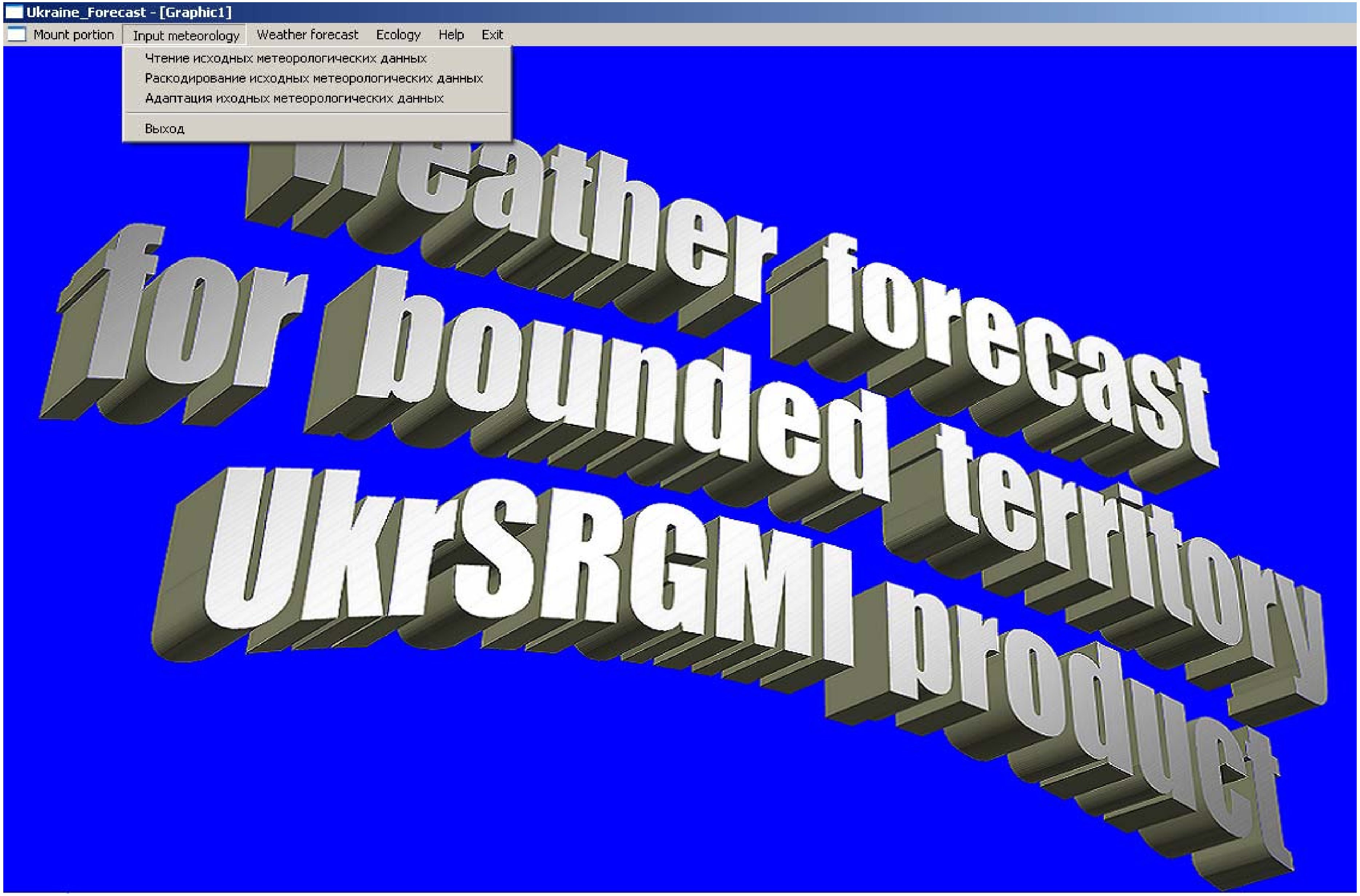
● given values, ——— development, ----- computational solution

PROGRAMMIC INTERFACE

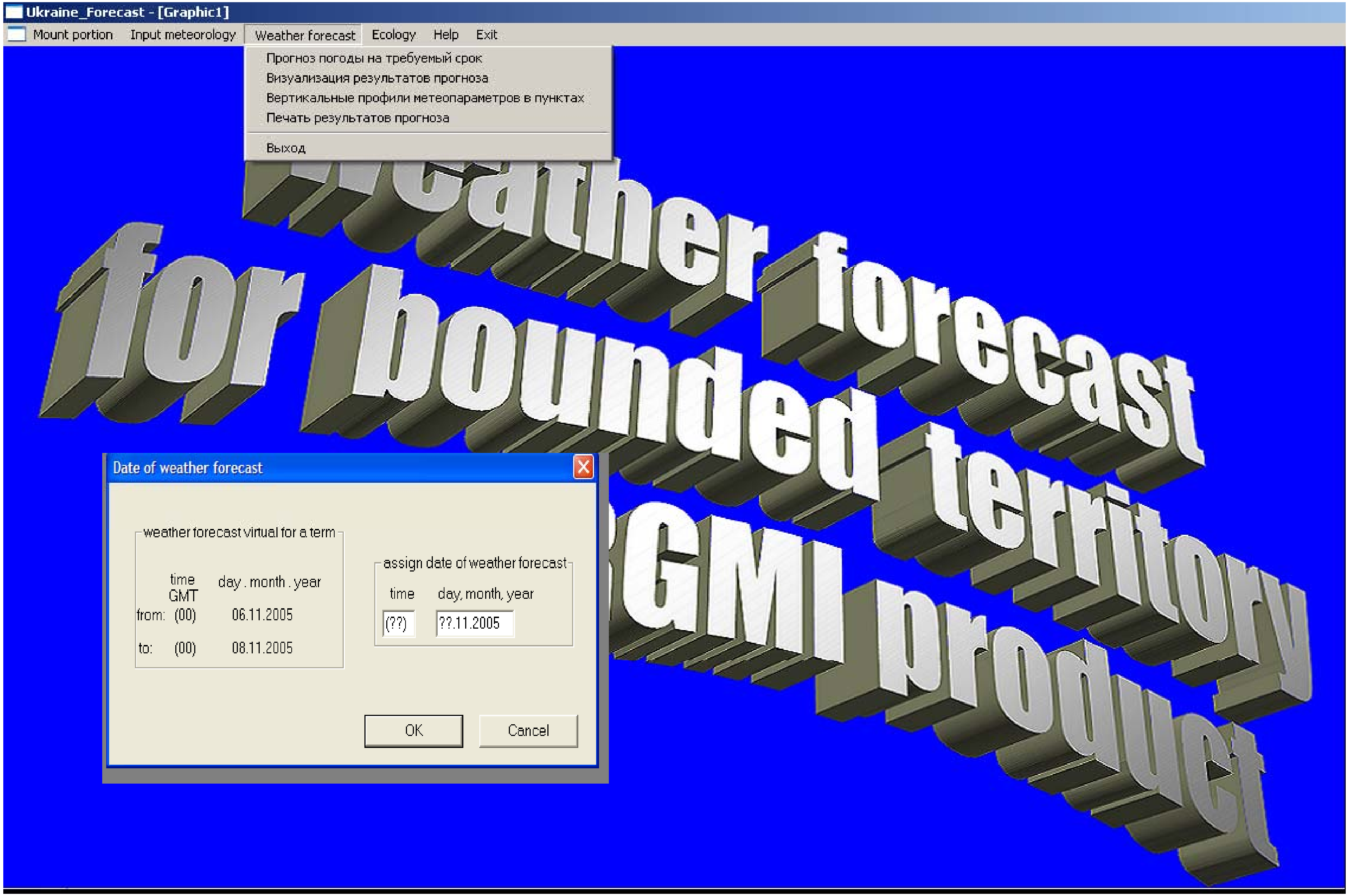
PROGRAMMIC INTERFACE



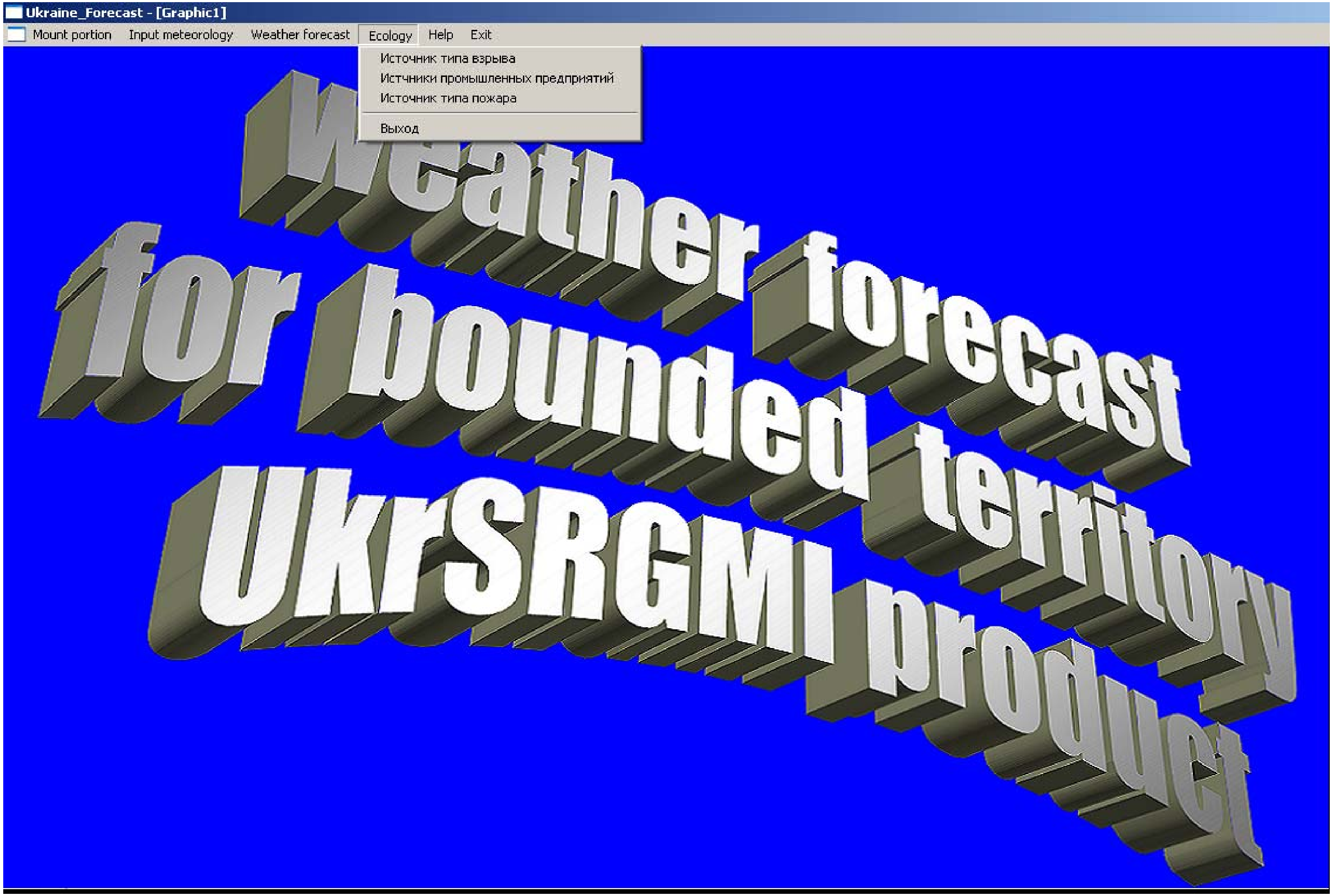
PROGRAMMIC INTERFACE



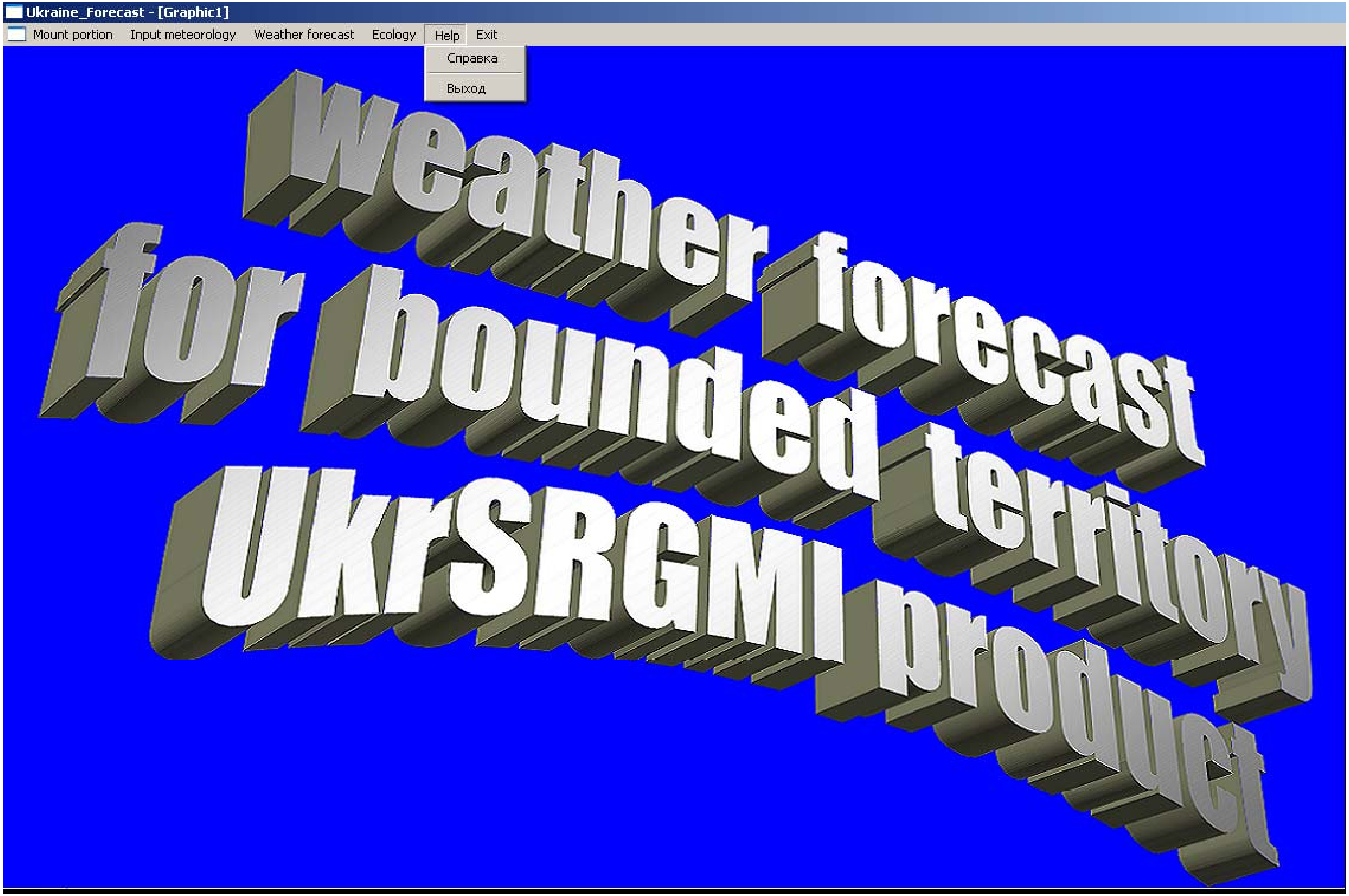
PROGRAMMIC INTERFACE



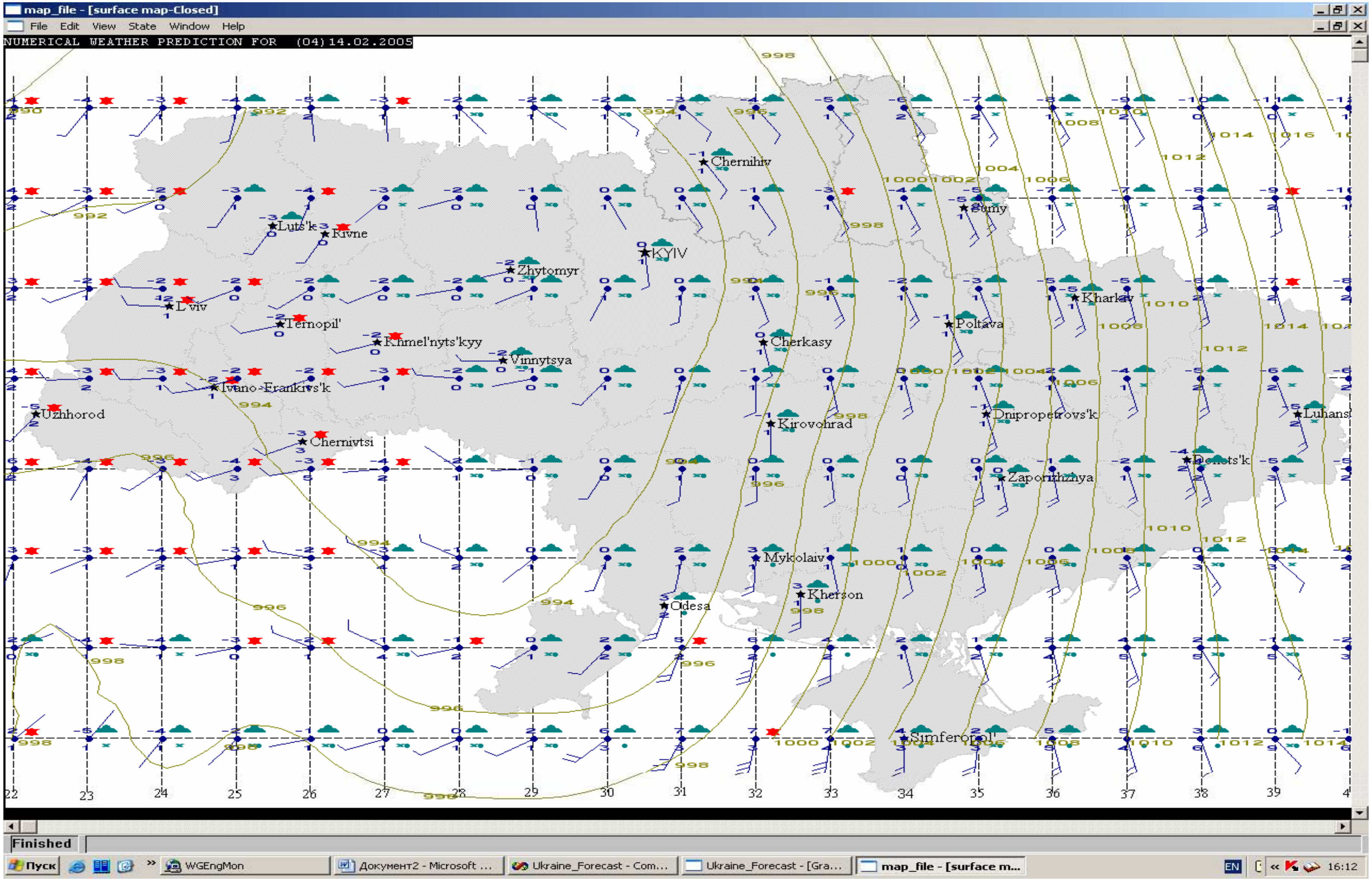
PROGRAMMIC INTERFACE



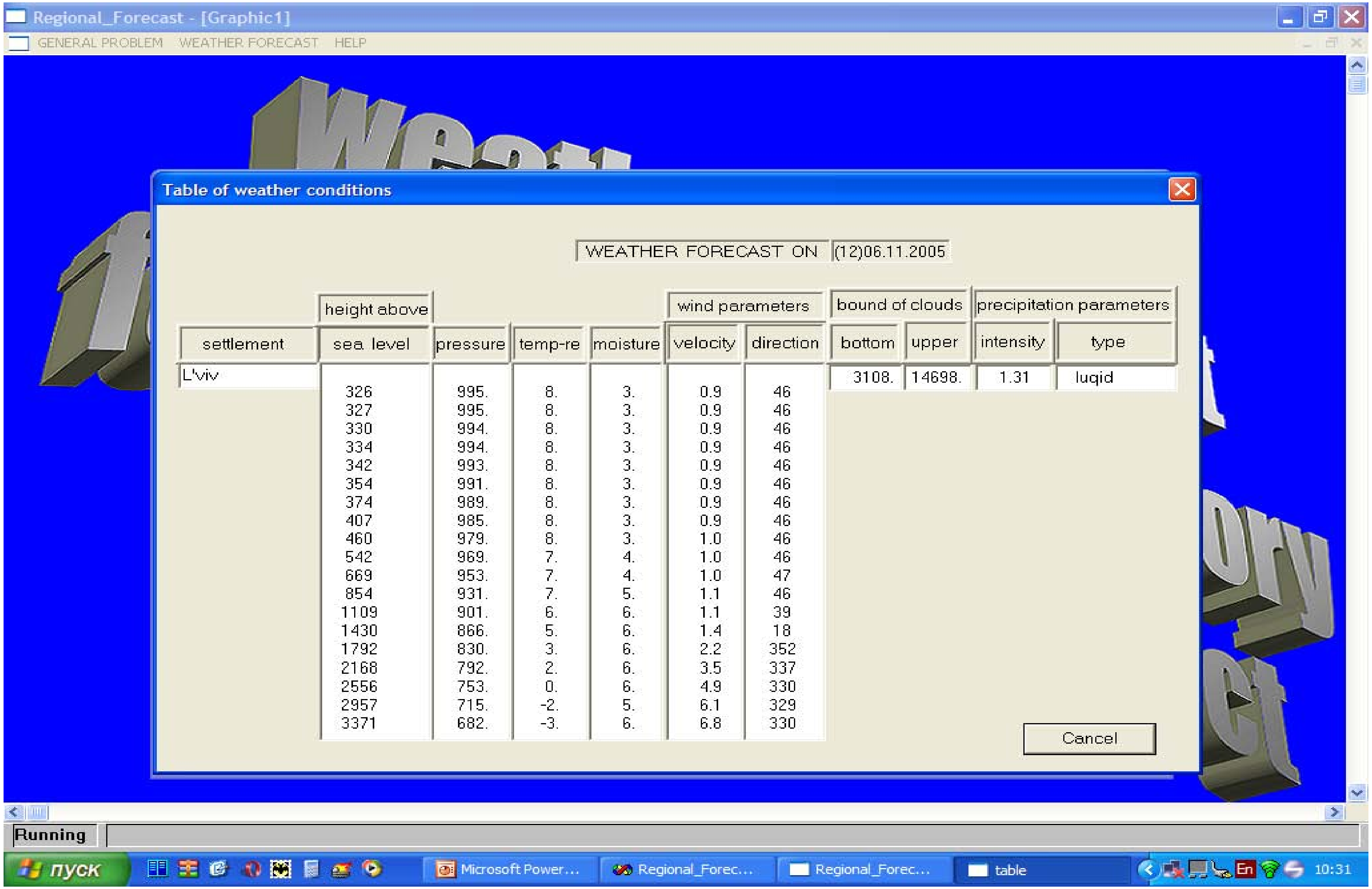
PROGRAMMIC INTERFACE



EXAMPLE WEATHER FORECAST



VERTICAL PROFILE OF WEATHER QUANTITY



settlement	height above sea level	pressure	temp-re	moisture	wind parameters: velocity	wind parameters: direction	bound of clouds: bottom	bound of clouds: upper	precipitation parameters: intensity	precipitation parameters: type
L'viv							3108.	14698.	1.31	luqid
	326	995.	8.	3.	0.9	46				
	327	995.	8.	3.	0.9	46				
	330	994.	8.	3.	0.9	46				
	334	994.	8.	3.	0.9	46				
	342	993.	8.	3.	0.9	46				
	354	991.	8.	3.	0.9	46				
	374	989.	8.	3.	0.9	46				
	407	985.	8.	3.	0.9	46				
	460	979.	8.	3.	1.0	46				
	542	969.	7.	4.	1.0	46				
	669	953.	7.	4.	1.0	47				
	854	931.	7.	5.	1.1	46				
	1109	901.	6.	6.	1.1	39				
	1430	866.	5.	6.	1.4	18				
	1792	830.	3.	6.	2.2	352				
	2168	792.	2.	6.	3.5	337				
	2556	753.	0.	6.	4.9	330				
	2957	715.	-2.	5.	6.1	329				
	3371	682.	-3.	6.	6.8	330				

ПРОГНОЗ РАСПРЕДЕЛЕНИЯ КОНЦЕНТРАЦИИ НЕКОТОРОГО ИНГРЕДИЕНТА С ПДК=0,000015